GW150914 Black Hole Merger Gravitational Wave Parameters Estimation

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Abstract

Energetic events such as black hole mergers generate gravitational waves detectable by the Laser Interferometer Gravitational-wave Observatory (LIGO). This study analyzes the LIGO gravitational wave data of the black hole merger event GW150914 to estimate parameters: chirp mass, mass ratio, individual black hole masses, luminosity distance, and geocentric time. Using the Bayesian inference and Markow Chain Monte Carlo (MCMC), four of the twelve event parameters were sampled. The inferred values align with those reported in official LIGO analyses, demonstrating that simplified Bayesian sampling can reasonably infer core merger properties using open-source gravitational wave data.

1. Introduction

Gravitational waves are oscillations in the gravitational field that travel at the speed of light. They were predicted by Einstein's general theory of relativity which states that massive accelerating objects distort spacetime and cause gravitational waves to ripple out from the source.

Black hole mergers are among the most energetic and commonly detected sources of gravitational waves in the universe. When a star significantly more massive than the Sun exhausts nuclear fuel in its core, the outward pressure that counteracted gravity is no longer present. The star then collapses under its own gravity.

If the core of the star exceeds 3 solar masses, it collapses into a black hole. Black holes are objects with an immense amount of mass concentrated in a very small space, resulting in an extremely strong gravitational pull. The escape velocity required to overcome this gravitational pull exceeds even the speed of light, and therefore black holes cannot be observed directly, but their properties can be studied by observing their effect on their surroundings or by analyzing gravitational waves produced during black hole merger events.

The features of a black hole merger are referred to as its parameters. These include properties such as individual black hole masses, chirp mass, spin magnitudes, tilts, right ascension, declination, distance to merger, orbital phase, etc. Gravitational waves carry crucial information about their source event's parameters.

To detect gravitational waves and study their properties, a highly sensitive instrument known as LIGO is used. There are two LIGO facilities in the U.S., one in Hanford, Washington, and the other in Livingston, Louisiana. Other gravitational wave detectors exist internationally, such as Virgo in Italy. Each detector has an L-shaped design with two arms that are 4 kilometers long each. At the point where the arms meet, a mirror splits a laser beam into both arms. Mirrors at the ends of the arms reflect the beams back and forth approximately 300 times to increase the distance traveled by the light. The beams of light are recombined when they meet. Gravitational waves stretch and squeeze space, so if a wave passes through the detector while the beams are in transit, it causes miniscule changes to the distance between the mirrors that cause an interference pattern when the beams are recombined. By analyzing this interference pattern, we can find the parameters of the event that the gravitational waves were caused by. The longer the distance that the light travels, the smaller the changes that LIGO can detect.

2. Method

Gravitational wave strain data recorded by the LIGO detectors is publicly accessible through the Gravitational Wave Open Science Center (GWOSC). The data is provided as a time series, showing the strain variation over time during gravitational wave events.

The strain data was used for parameter estimation via the Bayesian inference, a mathematical framework provided by Bayes' Theorem that updates prior knowledge about parameters based on observed data. The equation expressing the theorem is as follows:

$$P(F_{true}|D) = \frac{P(D|F_{true})\,P(F_{true})}{P(D)}$$

- $P(F_{true})$ is the prior distribution, representing existing knowledge or assumptions made about GW150914's parameters F_{true} before observing its gravitational wave strain data D. In this project, uniform priors were used with the following ranges and units:
  - Chirp Mass: 10-100 $M_{\odot}$
  - Mass Ratio: 0.5-1 (dimensionless)
  - Luminosity distance: 100-500 Mpc
  - Geocentric time: start time $\pm$ 0.1 s (GPS time)
- $P(D|F_{true})$ is the likelihood, representing the probability of D given F_{true}, weighted by the detector's power spectral density (PSD) to account for frequency-dependent noise.
- $P(D)$ is the total probability of D across all possible parameter values. It acts as the normalization constant to ensure all possible parameter probabilities sum to 1.
- $P(F_{true}|D)$ is the posterior distribution, representing the updated probabilities of F_{true} after considering D.

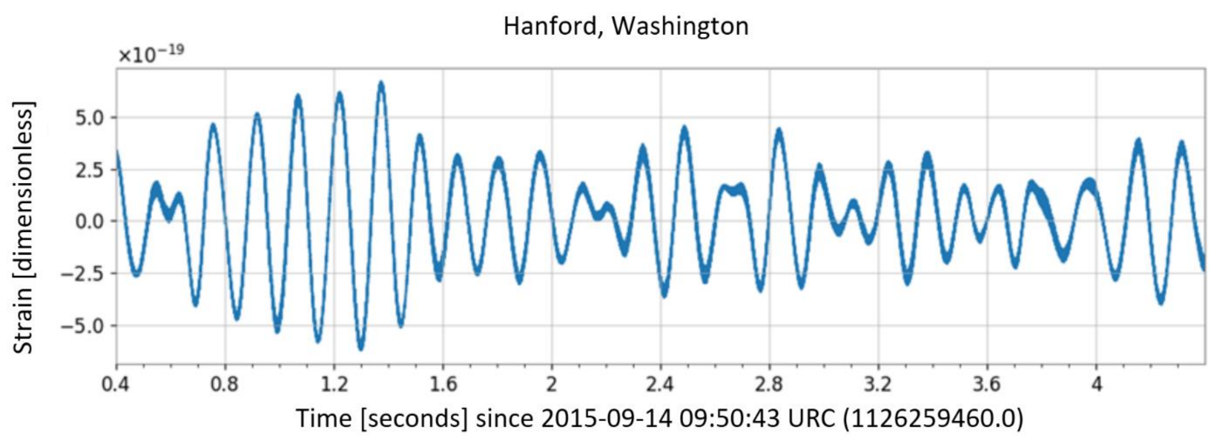
The posterior distributions were estimated using the MCMC method, a Bayesian statistical technique that samples from the prior distributions using Markov Chains. In MCMC, "walkers" are points that move through the parameter space by proposing new candidate points to move to.

A candidate point is more likely to get accepted if it has a higher probability than the current point. Walkers were initialized at arbitrary positions, and an initial burn-in phase of 500 steps was applied to reduce bias from the starting points. After burn-in, the walkers converged around the most probable parameter values within the range defined by the prior, producing the posterior distributions.

Sampling was conducted in Python using the Bilby package within a Google Colab environment. The nested sampler dynesty was used with 1000 live points, a stopping criterion of dlogz = 0.1, and the "unif" method for sampling. Parameters sampled included chirp mass, mass ratio, geocentric time, and luminosity distance. Spin and other extrinsic parameters were excluded from this simplified analysis. The individual black hole masses were derived from the sampled chirp mass and mass ratio using the following equations:

$$Chirp\ Mass = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}} \qquad Mass\ Ratio = \frac{m_2}{m_1}$$

3. Results



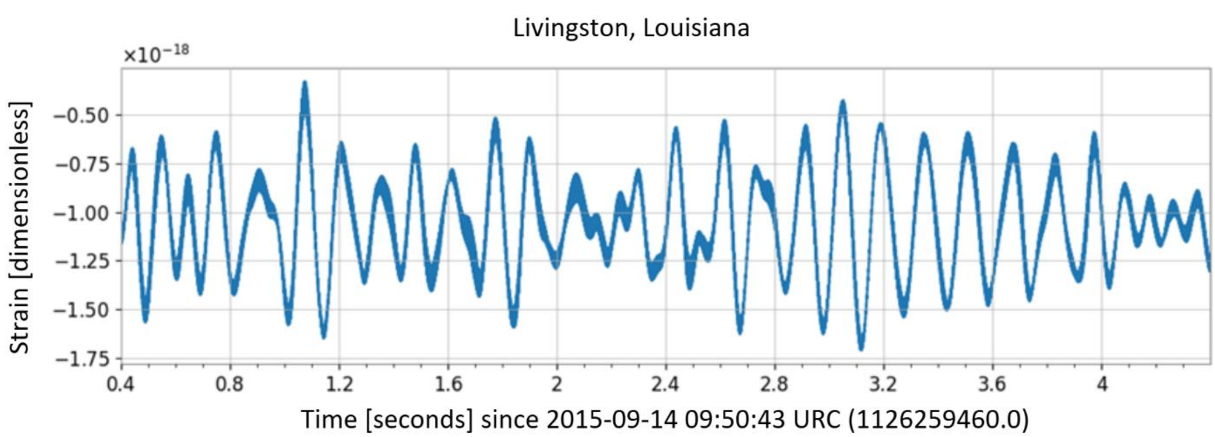


Figure 1. Gravitational wave strain versus time graphs for GW150914 measured by the two LIGO detectors. The maximum amplitude occurs at ~1.2 s after the referred coalescence time, corresponding to the merging of the two black holes.

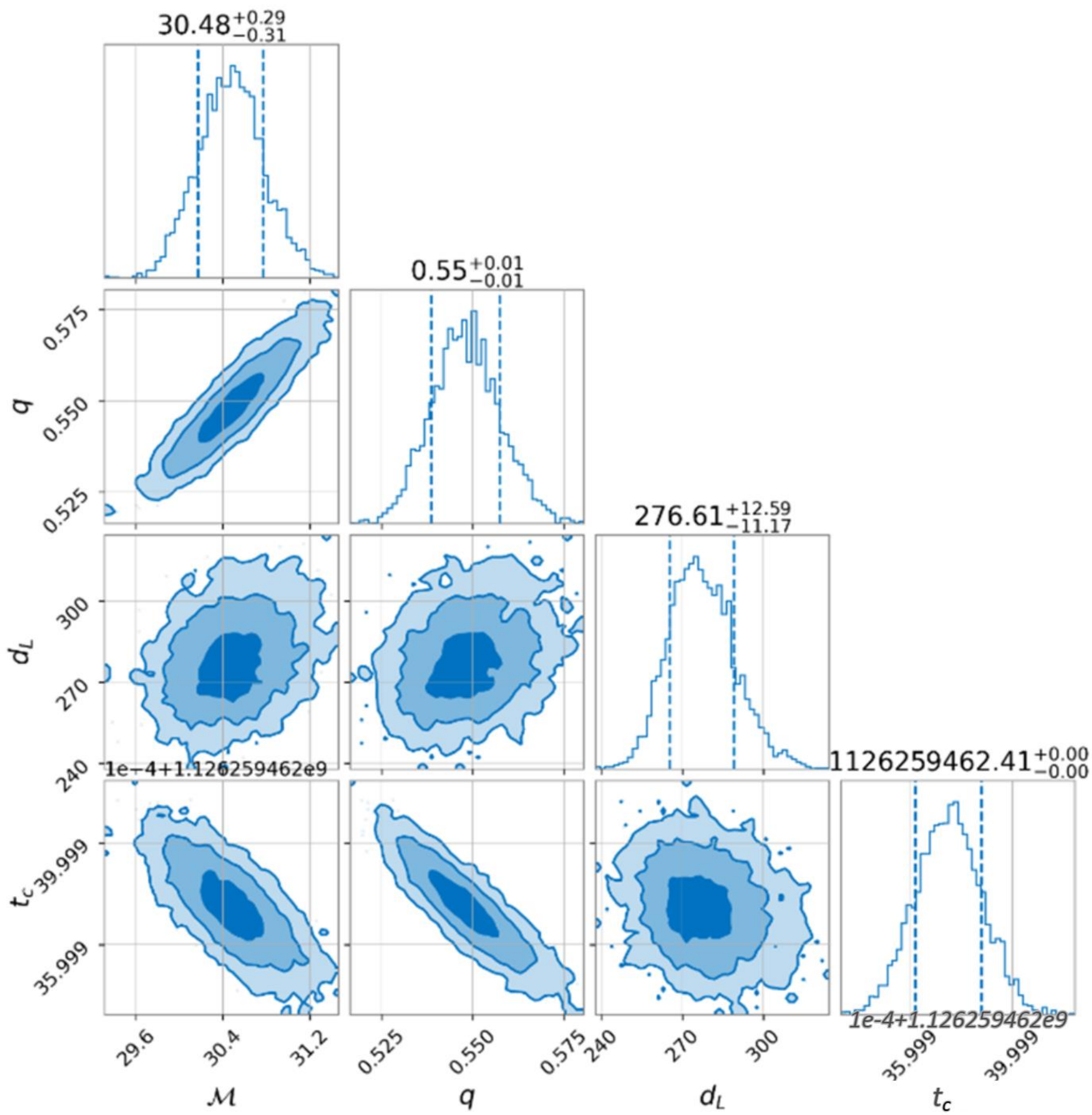


Figure 2. Corner plot visually representing the results from sampling. The histograms show posterior distributions of the sampled parameters (chirp mass M, mass ratio q, luminosity distance d_L, and geocentric time t_c). The peaks correspond to the most probable parameter values, while the spread reflects the uncertainties. Dashed vertical lines mark the uncertainty ranges. The contour

plots show the correlation between two parameters, with darker regions indicating higher probabilities.

Table 1.

Estimated parameter values alongside actual values reported in [10]

Parameter	**Estimated Value**	**Actual Value**
Chirp Mass ($M_{\odot}$)	$30.48^{+0.29}_{-0.31}$	$28.0^{+2.0\pm0.3}_{-1.7\pm0.3}$
Mass Ratio (Mass 2/Mass 1)	$0.55^{+0.01}_{-0.01}$	$0.82^{+0.17\pm0.01}_{-0.20\pm0.03}$
Mass 1 ($M_{\odot}$)	$47.71^{+1.04}_{-1.04}$	$35.8^{+5.3\pm0.9}_{-3.9\pm0.1}$
Mass 2 ($M_{\odot}$)	$26.15^{+1.03}_{-1.03}$	$29.1^{+3.8\pm0.1}_{-4.3\pm0.7}$
Luminosity Distance (Mpc)	$276.61^{+12.59}_{-11.17}$	$410^{+160\pm20}_{-180\pm40}$
Geocentric Time	1126259462.41	1126259462.4

4. Conclusion

This project successfully applies the Bayesian inference and MCMC sampling to LIGO gravitational wave data to estimate black hole merger parameters from GW150914. The results shown in Table 1 demonstrate that even when sampling for a limited number of parameters, the posterior estimates remain reasonably close to the published values, which were estimated using all parameters.

The main limitation arises from computational constraints. Although sampling for all 12 source parameters would yield more accurate estimates for parameter values, it is computationally infeasible within the scope of this project. Therefore, only 4 parameters were sampled for, while the remaining parameters were assigned fixed prior values.

Future projects on gravitational waves parameter estimation could benefit from upcoming missions like LISA (Laser Interferometer Space Antenna), which is a space-based gravitational wave detector scheduled for launch in the mid-2030s. It consists of 3 spacecrafts arranged in an equilateral triangle with side lengths approximately 2.5 million kilometers. Operating in the low-noise environment of space and with laser arms about 2,000 times longer than the distance traveled by the laser in LIGO, LISA will be capable of detecting a much wider range of gravitational wave sources. These include potential detections of supermassive black holes and the cosmic microwave background (CMB) radiation. Analyzing such detections would provide valuable insights into the early Universe and its formation, making LISA a major milestone in gravitational wave astronomy.

References

[1] Caprini, C., Brito, R., Franciolini, G., Nardini, G., Tamanini, N., & Steer, D. (2025). *Science of the LISA mission: A Summary for the European Strategy for Particle Physics*. https://indico.cern.ch/event/1439855/contributions/6461563/attachments/3045935/5381886/LISA_for_the_European_Strategy_of_Particle_Physics.pdf

[2] jakevdp. (2025). *Frequentism and Bayesianism: A Practical Introduction*. Github.io. https://jakevdp.github.io/blog/2014/03/11/frequentism-and-bayesianism-a-practical-intro/

[3] LIGO. (n.d.). *LIGO's Interferometer*. LIGO Lab | Caltech. https://www.ligo.caltech.edu/page/ligos-ifo

[4] LIGO Caltech. (2025). *What Are Gravitational Waves?* LIGO Lab | Caltech. https://www.ligo.caltech.edu/page/what-are-gw

[5] LIGO Scientific Collaboration. (2015). *Observation of gravitational waves from a binary black hole merger – LSC – LIGO Scientific Collaboration*. Ligo.org. https://ligo.org/science-summaries/GW150914/

[6] NASA. (2017, March). *Neutron Stars, Pulsars, and Magnetars - Introduction*. Nasa.gov; NASA. https://imagine.gsfc.nasa.gov/science/objects/neutron_stars1.html

[7] Navarro, D., & Perfors, A. (n.d.). *The Metropolis-Hastings Algorithm*. Retrieved August 12, 2025, from https://compcogsci-3016.djnavarro.net/technote_metropolishastings.pdf

[8] Pierce, R. (2017). *Bayes' Theorem*. Mathsisfun.com. https://www.mathsisfun.com/data/bayes-theorem.html

[9] Shivam Agrahari. (2021, July 27). *Monte Carlo Markov Chain (MCMC) explained | Towards Data Science*. Towards Data Science. https://towardsdatascience.com/monte-carlo-markov-chain-mcmc-explained-94e3a6c8de11/

[10] Abbott, B. P., et al. (2016, June 14). *Properties of the Binary Black Hole Merger GW150914.* Astrophysics Data System. https://ui.adsabs.harvard.edu/abs/2016PhRvL.116x1102A/abstract