

Exploring how Harmonic Structures Influence Human Emotional Perception of Music

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Abstract

For centuries, scientists and artists have been captivated by music's complex interplay of sound waves and harmonic structures. While physiological research has advanced our understanding of music's emotional effects, the precise mechanisms by which harmonic frequencies influence human emotion remain only partially understood. This paper investigates the interdisciplinary relationship between acoustics, auditory perception, and digital sound manipulation, offering new insights into how harmonic structures shape emotional responses. By integrating psychoacoustics with mathematical tools such as the Fourier Transform, this study examines how harmonic content is encoded by the auditory system, interpreted by the brain, and manipulated through digital technologies. The findings suggest applications in music production, therapeutic sound design, and emotionally adaptive audio systems.

Key Words: Harmonic structures, Fourier Transform, Auditory perception, Sound waves, Digital signal processing, Emotional sound design, Psychoacoustics

Introduction

Assuming a life expectancy of 80 years, an average person will spend approximately 10 of those years listening to music. This equates to nearly a sixth of a human's life dedicated to what could be fundamentally described as "organized sound" (U.S. Census Bureau, 2012). Music is more than just a form of entertainment; it affects us physically, emotionally, and psychologically. The significant time and resources humans dedicate to music, despite its unclear evolutionary benefits, have sparked investigations across fields like physics, physiology, and mathematics (Juslin & Västfjäll, 2008). Understanding music through an interdisciplinary lens reveals its profound emotional impact. Studying the physical and mathematical principles of sound and how it interacts with the auditory system helps us understand how the brain processes music. This approach not only enriches our understanding of perception and emotion but also has practical implications for therapeutic applications, music generation, and advanced sound design technologies (Disch & Edler, 2008; Menon & Levitin, 2005; Salimpoor et al., 2011).

Music is a physical phenomenon, with vibrations in a medium translated into sound waves. In its simplest form, music can be understood as the interaction of three fundamental elements: melody, harmony, and rhythm (McDermott & Oxenham, 2009). Yet, its impact goes far beyond these basic building blocks. Music triggers the release of dopamine in the brain, similar to the pleasure we experience from eating or social interactions (Patel, 2010). Music can evoke emotions regardless of its cultural or temporal context, suggesting that the key to its emotional impact lies within its harmonic structures. Recent findings reveal that harmonic frequencies are key to understanding the emotional appeal of music (Lostanlen, Andén, & Lagrange, 2019), raising an essential question: *How do sound waves and their harmonic frequencies influence human emotional responses to music?* Answering this question requires delving into the foundational components of sound and human perception.

This paper is organized into three major sections that highlight the relationship between sound, auditory perception, and human emotional experiences. First, it explores the physics of sound, examining how vibrations create harmonic waves and how the Fourier Transform mathematically decomposes them into their frequency components. Next, it delves into how our auditory system processes these signals, viewing the cochlea as a biological Fourier analyzer and exploring the brain's role in interpreting emotional responses. Ultimately, this paper connects these insights to real-world applications, including music production, sound design, and therapeutic uses, demonstrating how our emotional connection to music can enhance our well-being.

2. The Physics of Sound (Music as Soundwaves)

2.1 Fundamental Properties of Sound Waves

Sound waves are longitudinal mechanical vibrations that travel through a medium, such as air, water, or solid materials. Our auditory experience is directly influenced by the fundamental characteristics of these waves: amplitude, frequency, wavelength, and phase. Each factor significantly shapes our perception of sound's intensity, pitch, propagation, and how it interacts with its surroundings.

Amplitude represents the wave's energy and is translated into sound intensity, which our ears perceive as loudness. (Fong, 2024; Moossavi & Gohari, 2019). Amplitude is measured in decibels (dB). The greater the decibel, the louder a sound is perceived, and vice versa.

Frequency, measured in Hertz (Hz), defines pitch; the higher its frequency, the higher the pitch. The human auditory system typically perceives frequencies from 20 Hz to 20,000 Hz (McDermott & Oxenham, 2009).

Wavelength (λ) is the distance between two corresponding points of identical phase on a wave. It is inversely proportional to frequency and is calculated as:

$$\lambda = \frac{v}{f}$$

where v is the speed of sound in the medium, and f is the wave's frequency. As frequency increases, the wavelength decreases, and vice versa. For instance, low-frequency sounds have longer wavelengths, which allows them to travel farther and penetrate obstacles more effectively (Plaza Esteban, 2015).

Phase refers to the wave's position relative to time. Phase is a component crucial for determining how sound waves behave when interacted with, for example, through constructive or destructive interference. For example, noise-canceling headphones work due to phase manipulation of ambient noise (Lostanlen et al., 2019).

Harmonic waves are the foundation of music, and their main components include frequency and overtones. Overtones are essentially integer multiples of the fundamental frequency. The quality and quantity of these overtones collectively define the timbre of a sound. This is what allows us to differentiate between, for example, a violin and a flute playing the same note (Rakowski, 2009; Hailstone et al., 2009).

2.2 Elements of Music as Physical Phenomena

The relationship between sound waves and their harmonic components work together to form the main elements of music: melody, harmony, and rhythm. For example, melodies come from variations in pitch, harmony from the blending of multiple pitches and frequencies, and rhythm from the timing of sound and silence, such as the synchronized beats of a drummer.

Melody: Melody is a sequence of frequencies that vary over time, forming the most identifiable aspect of music. Humans often consider it “catchy” and what we remember from songs. Each pitch corresponds to

a specific frequency. The transitions between these pitches define the melodic contour, which conveys emotional nuance and meaning. Melodies frequently adhere to scales, structured sets of pitches based on frequency ratios, providing a piece's tonal framework.

Harmony: Harmony is the combination of multiple frequencies played simultaneously, creating chords in music. The differences between each frequency are called intervals, which humans perceive as consonant or dissonant. Consonant intervals create a sense of stability and resolution, while dissonant intervals sound unresolved and introduce tension. A frequency ratio describes the mathematical relationship between the vibrations of two pitches, for instance, the simplicity of a frequency ratio directly influences the classification of an interval as consonant or dissonant. For example, a perfect fifth, characterized by a simple frequency ratio of 3 : 2, produces a consonant and pleasing sound. In contrast, more complex ratios, like 16 : 15 ratio of a minor second, tend to generate dissonance, evoking a sense of tension that often seeks resolution within musical compositions (Lundqvist et al., 2009; Rakowski, 2009).

Rhythm: Rhythm is characterized by the arrangement of sound durations and silences over time, closely related to the periodicity of waveforms. This arrangement of time not only affects our emotional and physical reactions but also influences neural oscillations, which tend to synchronize with rhythmic patterns (Hailstone et al., 2009; McDermott & Oxenham, 2009). Additionally, the interplay between rhythm and frequency content enriches the complexity of our auditory experience.

2.3 Harmonic Frequencies

Harmonic waves are the foundation of music, with their primary components being the fundamental frequency and overtones. Overtones are integer multiples of the fundamental frequency, meaning they vibrate at frequencies that are whole-number multiples of the base pitch. The quality and quantity of these overtones define a sound's timbre, allowing us to distinguish, for example, a violin from a flute playing the same note (Rakowski, 2009; Hailstone et al., 2009).

To understand how these harmonic structures interact, mathematicians developed a powerful tool known as the Fourier Transform, which bridges the gap between time-domain and frequency-domain representations of sound. While we will explore the Fourier Transform more deeply in the next section, it is essential to note that this mathematical tool breaks down complex waves into simpler sinusoidal components. This decomposition is crucial for analyzing harmonic content and understanding how instruments produce distinct timbres (Lenssen & Needell, 2014).

This mathematical framework will be key in the upcoming discussion. We will use it to distinguish between harmonic structured sounds and the chaotic nature of noise.

3. Fourier Analysis and the Fourier Series in Sound Analysis:

3.1 Intro to Fourier Transform

As previously mentioned, musical sounds are composed of a fundamental frequency and a series of harmonics. To analyze these sounds effectively, we require a tool that reveals a sound's different overtones and underlying frequency structure: The Fourier Series. Fundamentally, the Fourier Series decomposes periodic functions into sums of sinusoidal components, allowing us to analyze their harmonic

content. Extending this concept further, the Fourier Transform connects the time and frequency domains of these frequencies, offering two complementary perspectives on sound.

But what do we mean by these “domains”?

A domain refers to the perspective from which we analyze a signal. In signal processing, there are two primary domains: the time domain and the frequency domain.

The **time domain** illustrates how a signal varies over time. For instance, when observing a waveform of a guitar note on an oscilloscope, you are viewing its time-domain representation, which displays how the signal's amplitude changes as time progresses (Learn EMC, n.d.) In this context, amplitude refers to the magnitude of a wave's oscillation, which directly corresponds to the signal's energy and perceived loudness. In music, higher amplitude values indicate greater sound intensity, while lower amplitude values represent softer sounds. Additionally, amplitude influences the sound's envelope, affecting how a note fades over time, which is crucial for distinguishing different instrumental timbres.

On the other hand, the **frequency domain** represents the signal in terms of its constituent frequencies, indicating the contribution of each harmonic component. While the time domain provides insight into how a signal evolves, it doesn't reveal the specific frequencies that compose the sound. This is where the frequency domain becomes essential, offering a different perspective that complements the time-domain view (Learn EMC, n.d.)

A thorough understanding of both the time and frequency domains is essential for a comprehensive analysis of signals, as they provide complementary insights into the characteristics and composition of sound. The Fourier Transform links these domains, enabling us to deconstruct a complex waveform into its constituent sinusoidal components, each corresponding to a specific harmonic frequency. In the following section, we will delve into the mathematical principles behind this process and examine how it uncovers the harmonic structure of musical signals.

3.2 Fourier Transform: Mathematical Breakdown

Basic sinusoidal functions serve as the foundation for all periodic waveforms. These functions repeat every 2π and correspond to pure tones with constant amplitude and frequency. However, musical signals are rarely this simple. They consist of multiple sinusoidal components, each vibrating at different frequencies. This complexity can be mathematically modeled using the Fourier Series:

Consider a basic *Sin* function:

$$f(x) = \text{Sin}(x)$$

For periodic signals, the Fourier series expresses the signal as a sum of sinusoids:

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

This equation shows that any periodic signal can be constructed by adding together multiple harmonic components where:

a_0 is the average value of the signal over one period (also known as the DC offset).

a_n and b_n are the amplitude coefficients of the cosine and sine waves, respectively, and determine the strength of each harmonic.

$n\omega_0$ is the frequency of the n -th harmonic (angular frequency)

Each term in the summation corresponds to a harmonic of the base frequency. For example, the first harmonic corresponds to the fundamental frequency, while the second harmonic vibrates at twice the frequency, and so on. (Lenssen & Needell, 2014)

However, while the Fourier series works well for periodic signals, most real-world sounds, such as those produced by musical instruments or the human voice, are non-periodic. To analyze these, we use the Fourier Transform, which generalizes the Fourier Series into an integral:

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

This integral breaks down the time-domain signal ($f(t)$) into its frequency components, where:

$f(t)$ is the signal in the time domain

$F(j\omega)$ is the representation of the signal in the frequency domain, indicating the amplitude and phase of each frequency component.

j : The imaginary unit where $j^2 = -1$

ω : The angular frequency, defined as $\omega = 2\pi f$, where f is the linear frequency in Hz.

$(j\omega)$: Together, $j\omega$ is used in the Fourier Transform to represent the oscillatory characteristics of the frequency components of a signal. (Freeman, 2023).

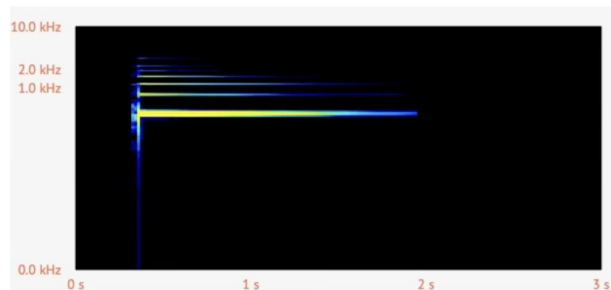
$e^{-j\omega t}$ is a complex exponential, which combines both sine and cosine components, thanks to Euler's identity $e^{j\theta} = \cos(\theta) + j \sin(\theta)$

This mathematical approach enables us to deconstruct complex sounds into their fundamental components, revealing the harmonic structures that define musical timbre. We will utilize this concept to examine the spectral composition of the same note played on different instruments. By investigating the harmonic content of this note on both instruments, we can see how the Fourier series allows us to visualize and analyze their unique timbral attributes.

3.3 Visual representation of varying Harmonic Components

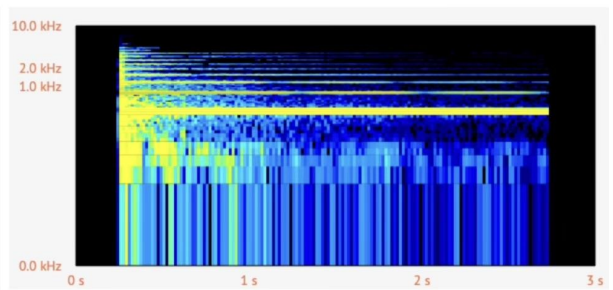
While musical notes share fundamental frequencies, their unique timbres are a direct result of variations in harmonic content. A guitar and a piano playing the same G4 pitch produce distinct spectral patterns due to their differing overtone structures and methods used in sound production. To analyze these differences, I recorded both instruments in Logic Pro and used Maztr's spectrum analyzer to generate spectrograms of the G4 note for each instrument, providing a visual representation of their harmonic structures. Fourier analysis was then applied to decompose each waveform into its frequency components, allowing for a direct comparison of overtone amplitude distributions.

Spectral Comparison of a G4 Note on Guitar and Piano



G4 On Guitar

Figure 1a.



G4 On Piano

Figure 1b.

Figure 1 illustrates the harmonic structures of a G4 note on both instruments. The guitar's spectrum (Figure 1a) displays a series of distinct harmonics with a faster energy decay (harmonics lose amplitude more quickly over time), resulting in a warmer and more percussive tone. In contrast, the piano's spectrum (Figure 1b) shows a richer harmonic distribution, with higher harmonic energy retention, producing a brighter and more resonant timbre.

Mathematical Comparison of Harmonic Content

By using the Fourier Series, detailed values of the Guitar and Piano's harmonic amplitudes were obtained. These values were manually recorded and compiled into a table for both visual and numerical analysis of their harmonic content.

Harmonic Frequencies and Amplitudes of a G4 note on Guitar and Piano

Harmonic Number (n)	Frequency ($n * f_0$)	Amplitude (Guitar)	Amplitude (Piano)
1st Harmonic	392 Hz	$A_1(G)=1.00$	$A_1(P)=1.00$
2nd Harmonic	784 Hz	$A_2(G)=0.6$	$A_2(P)=0.9$
3rd Harmonic	1176 Hz	$A_3(G)=0.4$	$A_3(P)=0.8$
4th Harmonic	1568 Hz	$A_4(G)=0.2$	$A_4(P)=0.7$
5th Harmonic	1960 Hz	$A_5(G)=0.1$	$A_5(P)=0.6$

Table 1. Harmonic Frequencies and amplitudes of a G4 note

Upon examining Figures 2a and 2b, we observe that the guitar spectrum displays approximately 5 harmonics, whereas the piano spectrum reveals 10 harmonics, particularly in the lower frequency range. This distinction plays a significant role in the overall fullness and sustain of the piano, compared to the guitar's sharp attack due to its reduced sustain. The piano's increased harmonic density can be attributed to its sound production mechanisms, where string vibrations are initiated by the percussive attack of the hammer keys, resulting in a longer sustain (Jancke, 2008; Rakowski, 2009). In contrast, the plucked nature of guitar strings leads to a quicker dissipation of harmonics, contributing to a warmer, percussive tone. Additionally, amplitude differences across harmonics further illustrate the timbre variations between the two instruments. As shown in Table 1, the guitar's harmonics show a faster decline in amplitude, meaning that its higher harmonics contribute less to the overall sound compared to the piano, where harmonics retain more amplitude across the spectrum. This difference explains why the piano produces a richer, resonant tone, whereas the guitar emphasizes its fundamental frequency with less harmonic sustain and content (Hailstone et al., 2009).

This comparison highlights how the Fourier Transform and the Fourier Series are fundamental tools for analyzing timbre. By breaking down a sound into its harmonic components, we can mathematically explain why the same musical note sounds different across instruments. The harmonic distribution and amplitude variation define the unique sonic fingerprint of each instrument, demonstrating how Fourier analysis bridges the gap between mathematical abstraction and auditory perception (Freeman, 2023). This concept reinforces the idea that music perception relies heavily on harmonic structures rather than isolated frequencies. Furthermore, the ability to distinguish between different timbres is essential for music perception, audio engineering, and digital sound synthesis, as harmonic structures are the key to human distinction between music and noise and the role of octaves in organizing sound perception.

4. Harmonic Structures, Noise, and the Role of Octaves

4.1 Harmonic Structures vs. Noise in Music

Although both music and noise are composed of sound waves propagating through a medium, such as air, their key distinction lies in the organization of frequencies. Musical notes consist of harmonic structures: frequencies that follow simple and predictable relationships, integer multiples of a fundamental

frequency, as mentioned in section 2.3. In contrast, noise consists of non-harmonic, unpredictable frequencies, lacking any coherent pattern, which is why the human ear perceives it as “chaotic” (Freeman, 2023)

Fourier analysis provides a mathematical method for distinguishing between harmonic structures, such as music, and noise. When a Fourier transform is applied to a musical tone, it shows a structured arrangement of harmonically related frequencies. On the other hand, noise produces a random spectrum of frequencies with no clear structure. (Freeman, 2023)

This distinction is critical in fields like audio engineering, where separating harmonic sound from background noise is the foundation of sound quality optimization. Digital sound processing relies on the Fourier series to isolate, enhance, or suppress different frequency components, demonstrating the importance of understanding and differentiating harmonic structures in music (Freeman, 2023). Figure 2 illustrates the visual contrast between noise and tone, highlighting how harmonic structures in a pure tone differ from the noise, which lacks a clear pattern.

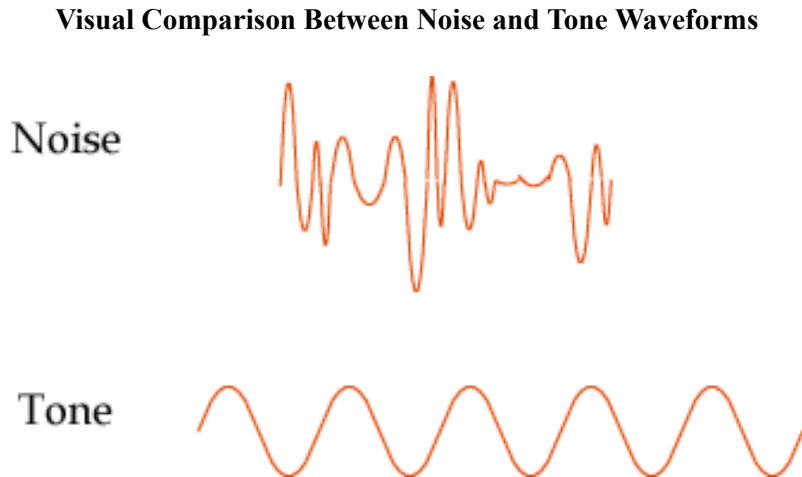


Figure 2. Visual comparison of noise and tone waveforms.

The upper waveform represents noise, which is irregular and lacks a consistent pattern due to random, non-harmonic frequencies. In contrast, the lower waveform represents a tone, showing smooth, periodic oscillations that reflect a stable harmonic structure. (Adapted from Music Ability Lessons, 2023.)

4. The Physics of Octaves and Frequency Doubling

An octave is a fundamental musical interval characterized by a 2:1 frequency ratio, meaning that its octave vibrates at exactly twice the rate for every cycle of vibration of a lower-pitched note. This doubling causes the waveform of the higher note to align periodically with that of the lower note (McDermott & Oxenham, 2009), reinforcing the perception of a structured and stable relationship between them. For example, an A₃ note has a frequency of 220 Hz, while its octave (A₄) has a frequency of 440 Hz, which

we recognize as the same pitch at a higher register. They're the same note, just one octave apart. This reveals that our perception of pitch is not solely based on absolute frequency but on the relative relationships between frequencies (McDermott & Oxenham, 2009). This principle is foundational to musical organization and human auditory perception, allowing us to interpret melodies and harmonies consistently across different registers.

The harmonic series, a natural phenomenon in vibrating systems, directly results in frequency doubling, which forms the foundation of octave perception in music. As previously mentioned, the second harmonic always represents the octave above the fundamental pitch, making the octave a universal principle in musical structure (McDermott & Oxenham, 2008). This property supports the widespread use of octave-based structures in music, particularly in systems like the 12-tone equal temperament scale (Deutsch, 2013). Because the harmonic series naturally produces octaves through frequency doubling, musical scales have evolved to align with the way the human auditory system organizes pitch (Jourdain, 1997). The 12-tone system, in particular, reflects this organization by dividing the octave into equal steps, reinforcing our perception of harmonic relationships across registers (Freeman, 2023). Octave equivalence refers to the auditory phenomenon in which tones an octave apart are perceived as nearly identical by listeners. This perception is consistent across instruments, cultures, and tuning systems, suggesting that the brain instinctively groups these pitches as functionally the same, regardless of their absolute frequency (Deutsch, 2013).

Additionally, the inherent nature of octaves explains why musical instruments, despite their varied timbres and harmonic overtones, produce structured and harmonious sounds. This natural frequency relationship not only shapes the way instruments generate sound but also influences how the brain organizes and interprets auditory input. The way our auditory system processes octave relationships reflects an underlying efficiency in perception, which extends beyond music and into how we compress and categorize sound (Rakowski, 2009). This effective octave-based structure goes beyond music, it influences cognitive processes like memory and categorization. This topic is further explored in section 5.

4.3 Octaves as a Natural Compression System

The role of octaves extends beyond musical theory. Octaves are directly linked to the brain's ability to process auditory information efficiently. The human auditory system does not perceive each frequency as an isolated individual component but rather organizes sounds into harmonic categories based on relationships like octaves and fifths. These relationships allow the brain to compress auditory information by categorizing multiple frequencies as perceptually similar or functionally equivalent (Rakowski, 2009). This is particularly evident in octave equivalence, where tones that differ by a 2:1 frequency ratio are interpreted as the "same" note at different registers. The brain prioritizes relative pitch relationships (how frequencies relate to one another) over absolute frequency values. As a result, melodies remain recognizable even when transposed to different keys and registers, and the proportional structure of their intervals stays intact (Okazaki, 2015).

Musical tuning systems are cultural extensions of this perceptual organization. Two prominent systems used to define pitch relationships are equal temperament and just intonation. Equal temperament, the standard in modern Western music, divides the octave into twelve logarithmically equal semitones, meaning that each step increases in frequency by the same constant ratio, rather than by the same absolute

number of Hertz (Hinrichsen, 2015). Each pitch is derived by multiplying the previous note's frequency by the twelfth root of 2 ($\sqrt[12]{2}$). This structure enables instruments to modulate between keys while maintaining consistent interval sizes, which is essential for Western harmony (Hinrichsen, 2015). However, by equal temperament having these uniform frequency ratios, some intervals lose purity/consonance compared to other tuning systems, such as intonation (EarMaster, n.d.).

By contrast, just intonation tunes intervals using precise whole-number frequency ratios (meaning that the frequencies of notes are simple mathematical multiples of one another, resulting in acoustically pure and naturally consonant intervals) drawn directly from the harmonic series. For example, a perfect fifth follows a 3:2 ratio, and a major third follows a 5:4 ratio. These ratios produce purer and more resonant harmonies; however, they make modulating keys more complex as the intervals are not evenly distributed throughout the octave. (Okazaki, 2015, p. 16).

Figure 3 shows harmonic intervals above a 110 Hz base pitch (A2), including octave (2:1), perfect fifth (3:2), perfect fourth (4:3), and major/minor thirds (5:4 and 6:5), with associated tuning differences (in cents) compared to equal temperament.

Common Harmonic Ratios in Just Intonation (String tuned to A 110Hz)

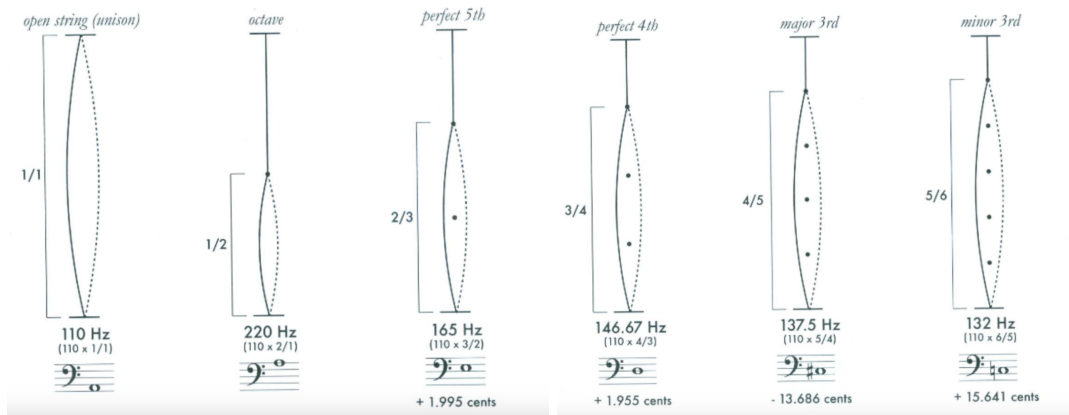


Figure 3. The relationship between pitch and the location of a stop on a string tuned to A 110 Hz (Okazaki, 2015, p. 16).

While just intonation aligns more closely with natural harmonic patterns, it is not globally used. Other tuning systems (Such as the Pythagorean, meantone, and non-Western temperaments) exist across different cultures and musical traditions. However, this paper focuses on equal temperament and just intonation because they are the most commonly used tuning systems in modern and historical music, especially within Western traditions. Their direct connection to harmonic ratios and the physics of frequency organization makes them particularly relevant to this investigation.

Ultimately, the brain's ability to organize and simplify pitch into octave-based categories demonstrates a broader cognitive pattern, which is simplifying complexity through structure. This efficiency supports not only music perception but also memory, pattern recognition, and emotional understanding. These concepts lay the foundation for the upcoming section, which examines how these harmonic relationships are biologically processed by the auditory system.

5. Auditory Mechanics and the Psychological Perception of Sound

5.1 The Auditory System: The Physiological Extent of Harmonic Components

Figure 5 presents a visual representation of the auditory system, emphasizing the structures of the outer, middle, and inner ear, which are involved in the process of auditory signal transmission. This diagram introduces the anatomical pathway through which sound waves travel before being converted into neural signals by the cochlea. This section explores how understanding this physiological foundation is essential for analyzing how harmonic sound components are processed and interpreted by the brain.

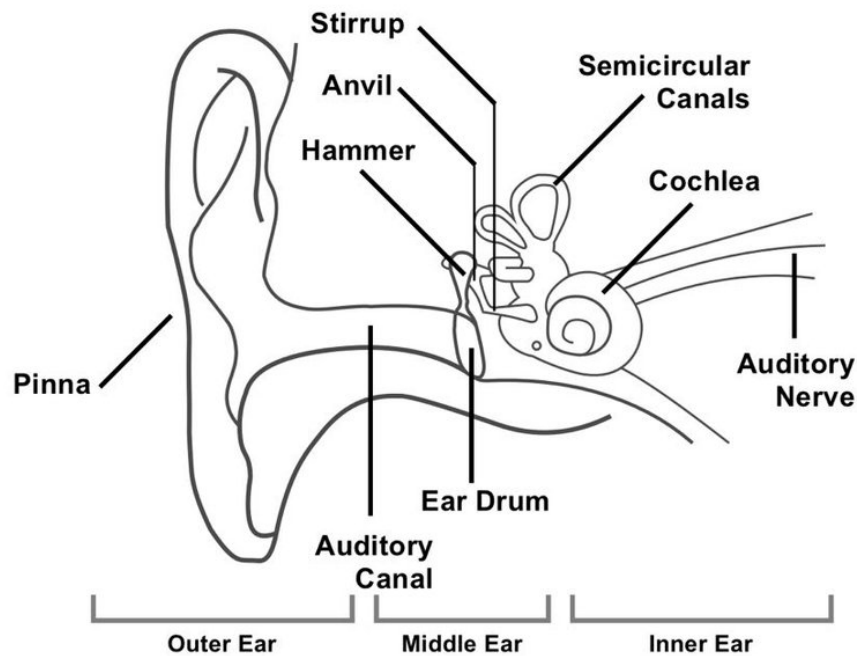


Figure 4, Diagram of the human ear, showing the outer, middle, and inner ear structures involved in auditory signal transmission. The cochlea and auditory nerve, located in the inner ear, are responsible for converting mechanical sound waves into neural signals. (Adapted from Purves et al., 2001).

The auditory system transforms mechanical sound waves into neural signals that the brain interprets as sound. This process begins when sound waves, and especially their frequency, amplitude, and timbre components, travel through the ear canal and vibrate the tympanic membrane (also known as the eardrum). These vibrations pass through the ossicles, a series of small bones located in the middle ear, and ultimately reach the cochlea, a complex fluid-filled and spiral-shaped structure within the inner ear responsible for converting sound into electrical signals.

Inside the cochlea, the basilar membrane serves as a biological frequency analyzer, with different frequencies affecting specific regions. Higher frequencies stimulate regions closer to the base of the cochlea, while lower frequencies reach the apex (Gold et al., 2019). This arrangement allows the breakdown of complex sound waves into their various harmonic components, much like the Fourier Transform mathematically decomposes waveforms into their specific frequencies. This natural spectral decomposition forms the foundation of how we perceive pitch and timbre (Alahmad et al., 2021).

Once converted into neural signals, these frequency patterns travel to the auditory cortex of the brain, where neurons synchronize their activity with incoming frequency patterns, forming the psychological perception of pitch (Gold et al., 2019). During this stage, the brain not only identifies frequencies but also interprets harmonic structures, allowing us to differentiate between instruments, voices, and musical intervals (Lee & Cheng, 2020). These processes ultimately contribute to our cognitive and emotional engagement with sound, a concept that is explored further in later sections.

5.2 The Ear as a Fourier Analyzer

As previously mentioned, the human auditory system is inherently designed to analyze sound frequencies, functioning in a way similar to the Fourier Transform. While the Fourier Transform mathematically decomposes complex waveforms into their constituent sinusoidal frequencies, the cochlea biologically breaks down incoming sound waves and distributes their frequency components along the basilar membrane (Alahmad et al., 2021). This process is known as tonotopic mapping, and it occurs when different sections of the basilar membrane respond to distinct frequency ranges. Then, the auditory nerve encodes this information and transmits it to the auditory cortex, where harmonic structures are reconstructed and interpreted as musical pitches, speech, or environmental noise (Lee & Cheng, 2020).

Neural processing further refines this spectral analysis through phase-locking, where neurons synchronize their firing to the phase of a sound wave, preserving frequency details (Cariani & Delgutte, 1996). This precision is important for pitch perception and for distinguishing between similar sounds. In music, harmonic relationships that share simple frequency ratios, such as perfect fifths (3:2) or octaves (2:1), are more easily perceived and categorized due to the brain's internal grouping mechanisms, as discussed in Section 4.3 (Skoe & Kraus, 2010). Here, a ratio refers to the numerical relationship between two frequencies (e.g., 300 Hz to 200 Hz is a 3:2 ratio), while an interval describes the musical distance between two notes on a scale, such as a perfect fifth or octave. This distinction ultimately reinforces our perception of consonance and musical structure.

The ear's ability to analyze sound frequencies plays a crucial role in both musical perception and speech processing, supporting our ability to interpret pitch, tone, and language. Additionally, the auditory system's biological interpretation of sound also helps in spatial hearing and the analysis of complex auditory environments, allowing a complex auditory experience. (McDermott & Oxenham, 2009). Figure 5 illustrates these biological mechanisms by showing how the cochlea is tonotopically organized. Different regions of the cochlea correspond to different frequency ranges, enabling the brain to process pitch information based on where sound vibrations trigger the basilar membrane (Moerel, De Martino, & Formisano, 2014)

Tonotopic Mapping of the Cochlea

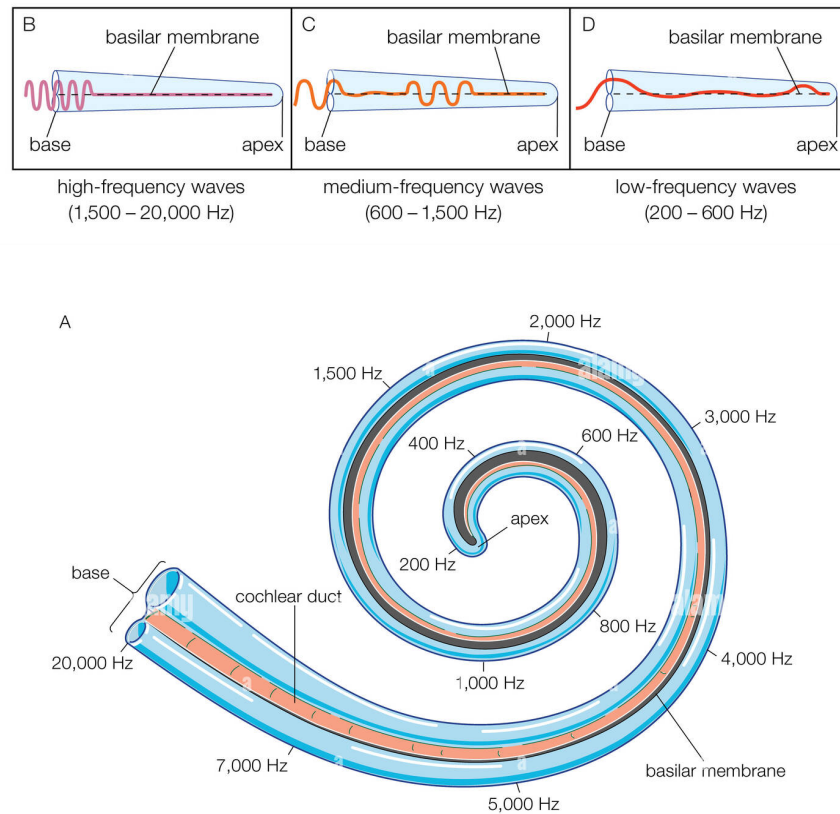


Figure 5. Tonotopic mapping of the cochlea and basilar membrane vibrations, showing how different frequencies stimulate specific regions along the membrane. High-frequency sounds activate the base, while low-frequency sounds reach the apex, allowing the cochlea to act as a biological frequency analyzer. (Adapted from Alamy Stock Photo, 2022.)

5.3 The Brain's Role in Connecting Music to Emotion

The human brain not only categorizes music as “organized sound” by breaking down its harmonic structures, but it also interprets these sounds, stimulating significant emotional responses. Once frequencies are interpreted by the ear and transmitted through the auditory nerve, the information reaches the auditory cortex in the temporal lobe. The auditory cortex is responsible for identifying pitch and timbre as well as recognizing familiar musical intervals or melodic patterns (McDermott & Oxenham, 2008). However, the brain's role in sound processing goes beyond harmonic recognition, and it extends to the emotional interpretation of auditory stimuli, connecting these musical features to subjective individual experience and memory. This process links the auditory system to other areas of the brain, including the limbic system, as shown in Figure 6.

The Limbic System

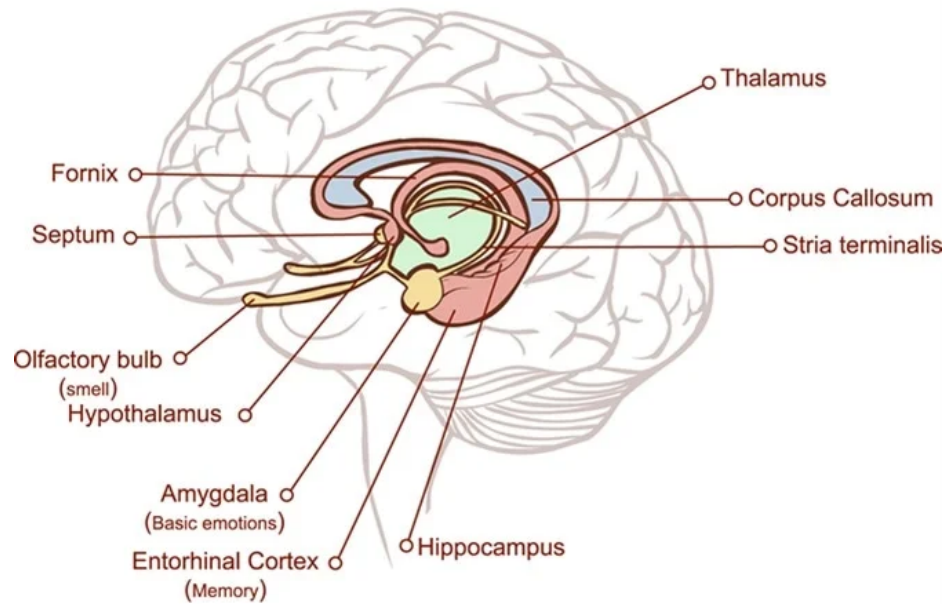


Figure 6: The limbic system and its key structures involved in emotional processing. Adapted from News-Medical.net (2023).

The limbic system includes structures such as the amygdala, which evaluates the emotional salience of sensory input, and the hippocampus, which connects stimuli to autobiographical memory. These two structures play the most direct and important roles in processing music-related emotions as they directly evaluate emotional meaning and connect it to memory. In contrast, other regions like the hypothalamus and thalamus play broader roles in emotional regulation and are less specifically linked to auditory and musical analysis, which is why they will be discussed less in this paper.

After initial acoustic processing occurs in the auditory cortex, key features of the sound, such as harmonic tension, resolution, or timbral color, are transmitted to the amygdala (Koelsch, 2014). The amygdala then “assesses” the emotional relevance of these patterns by comparing them to previous emotions based on prior experiences and instinctual survival cues. This evaluation determines whether the music should trigger an emotional reaction and what kind. If the harmonic sequence resembles patterns previously associated with safety, reward, or sadness, the amygdala categorizes it as meaningful. Simultaneously, the hippocampus may activate to collect memories tied to similar musical experiences, deepening the listener’s personal and emotional response (Janata, 2009). For instance, a soft piano melody may remind one listener of a moment of grief, while it may trigger feelings of nostalgia or joy in another listener. These evaluations show why music can trigger differing strong, and personal emotional responses in different people, even when the structural elements of the music remain the same for all listeners.

Beyond these evaluations, music also engages the brain’s predictive coding system. This system allows the auditory cortex to anticipate harmonic progressions based on familiarity and prior exposure. For example, when a musical phrase resolves in an expected way, such as a dissonant chord resolving into a consonant chord, the brain registers this structural fulfillment as emotionally satisfying

(Koelsch et al., 2013). These harmonic “resolutions” are not just recognized but expected, resulting in the activation of the brain’s reward system.

The Reward System

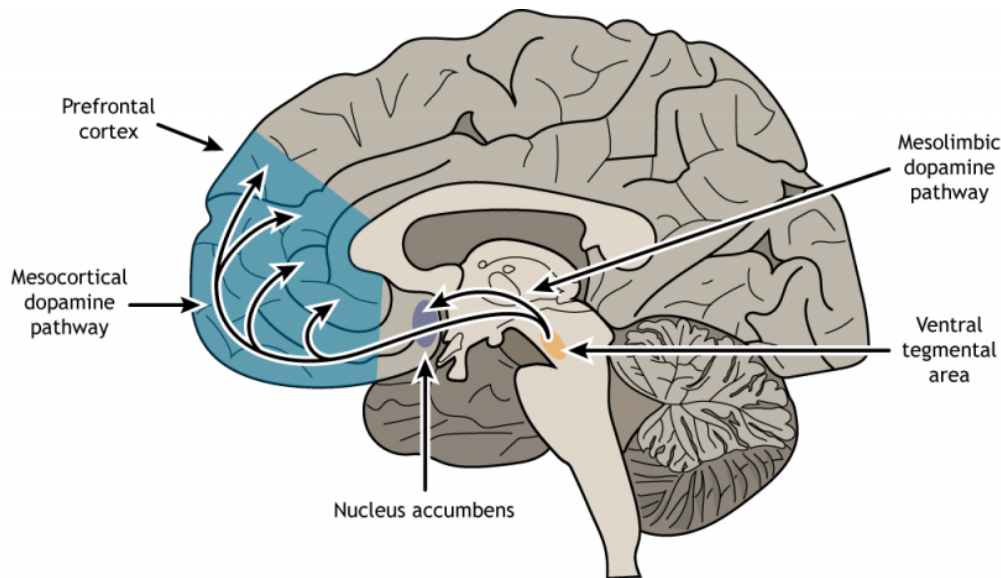


Figure 7. Neural reward system involved in musical emotion, including the VTA, nucleus accumbens, and prefrontal cortex. Adapted from Open Neuroscience Textbook (MSU, 2021).

The reward system is a network of regions responsible for motivation, reinforcement, and pleasure. When harmonic expectations are met or exceeded, specific neurons in the ventral tegmental area (VTA) release dopamine into the nucleus accumbens and prefrontal cortex, generating sensations of reward and emotional pleasure (Salimpoor et al., 2011; Zatorre & Salimpoor, 2013). Dopamine, a neurotransmitter associated with motivation and emotional learning, reinforces the neural pathways linked to pleasurable experiences, strengthening future engagement with similar stimuli. In the context of music, the brain treats tonal resolution, rhythmic build-up, and harmonic tension as emotionally significant experiences, creating a feedback loop that encourages repeated listening (Menon & Levitin, 2005). Music acts as a form of emotional reinforcement, using structure and tension-resolution patterns to trigger a response typically associated with basic survival behaviors such as eating or social bonding. This process is illustrated in Figure 7, which highlights the core components of the brain’s reward system involved in musical pleasure.

It is important to mention that the degree of reward triggered by musical stimuli can vary significantly between individuals. While the underlying neural mechanisms, such as activation of the limbic and reward systems, are shared across humans, the specific emotional impact of music is shaped by factors like cultural exposure, personal associations, and neurological sensitivity (Koelsch, 2014). For example, musical intervals or timbres considered consonant in one culture may be perceived as unfamiliar or emotionally neutral in another, highlighting how context and learning influence the emotional interpretation of music.

Together, the auditory cortex, limbic system, and reward system form an interconnected neural network that interprets, evaluates, and reinforces emotional experiences in music. This system does not passively receive sound. Instead, it actively decodes, predicts, and assigns meaning to harmonic information. In doing so, it mirrors the logic of Fourier decomposition: breaking complex musical signals into distinct components that can then be emotionally processed. In the case of the brain, however, this decomposition is not just mathematical; it is psychological, biological, and profoundly human.

The auditory cortex, limbic system, and reward system work together as a cohesive neural network that turns harmonic sound into emotional experiences. The brain's reaction to music is a complex process that relies on both biological functions and personal experiences. It begins with the decomposition of sound frequencies in the cochlea, moves through the limbic system's emotional assessment, and is finally followed by dopamine release from the reward system, shaped by musical expectations and individual characteristics.

This physiological pathway is comparable to the mathematical structure of Fourier analysis. While the brain does not perform a Fourier Transform, the cochlea functions similarly by decomposing incoming sound waves into their frequency components, with the different regions of the basilar membrane responding to specific frequencies. However, unlike mathematical Fourier decomposition, the brain's interpretation of frequencies is shaped by biological constraints, memory, and emotion. In this way, Harmonic patterns are not only decoded but interpreted, predicted, and rewarded, making music a powerful and subjective personal stimulus.

The following sections will explore how this biological and mathematical connection applies to modern sound technology. From compression algorithms to audio synthesis, these principles, rooted in Fourier analysis, allow modern music to be manipulated, edited, and refined, making the music-listening experience increasingly more complex and, with it, its emotional impact.

6. Modern Digital Tools for Harmonic Manipulation

6.1 Advances in Sound Engineering and Harmonic Synthesis

The application of harmonic decomposition in modern digital sound engineering has transformed how we create, manipulate, and perceive music. Just as the human auditory system separates frequencies, digital tools replicate these processes to analyze and reshape sound. At the core of these advancements are digital audio workstations (DAWs), software synthesizers, and spectral editing platforms, all of which rely on mathematical frameworks such as the Fourier Transform to isolate and manipulate individual frequency components within a complex sound wave. This is used in music production, sound design, and sound engineering.

A Digital Audio Workstation, or DAW, is any music software used for recording, editing, and producing audio. It allows users to manipulate sound in both the time domain and the frequency domain through a graphical user interface (GUI), a visual layout that displays waveforms, controls, and tools for editing, making complex processes accessible. Popular DAWs like Ableton Live, Logic Pro, and FL Studio enable precise sound manipulation through built-in tools such as EQs and spectral analyzers, which are mainly based on Fourier analysis. With the use of plug-ins, which are modular extensions that perform tasks such

as spectral analysis or sound synthesis, these extensions isolate specific harmonics or frequencies, which allow timbre altering, balance adjustment, and the use of effects such as equalization (EQ), reverb, delay, distortion, and modulation.

The use of electronic instruments, particularly synthesizers, has become increasingly common in modern music due to their versatility in shaping tone and texture. Synthesizers create sound by manipulating audio signals. One of the most useful types of synthesizers is the additive synthesizer, which layers multiple sine waves, each representing a specific frequency. Synthesizers that use additive synthesis resemble the principles of the Fourier series by adding specific harmonics to create more complex sounds. Individual sine wave components (harmonics) are combined to recreate any periodic waveform. By adjusting each harmonic's amplitude, phase, and frequency, musicians can precisely control timbre and design textures that evoke specific emotional or atmospheric effects.

For example, additive synthesis is especially popular in ambient, electronic, and cinematic genres, where sound designers often create different sounds and textures by layering and shaping individual harmonics. These techniques are commonly used in digital synthesizers and plug-ins available in modern DAWs.

This synthetic manipulation of harmonic content directly connects to how the cochlea processes sound. Just as the cochlea breaks down real-world sounds into their frequency components along the basilar membrane, additive synthesis constructs these same components. Timbre, a key feature of music our auditory system uses to differentiate sounds regardless of pitch, significantly influences our emotional experiences. A study by Siedenburg et al. (2023) found that changes in harmonic content can evoke different emotional responses, even when the pitch remains unchanged. This study highlights harmonic manipulation's important role on emotional engagement with music.

Overall, Modern digital tools like DAWs, additive synthesizers, and spectral editors do more than just enhance creative expression, they use processes similar to the auditory system's natural functions of harmonic analysis to achieve desired sounds. By utilizing mathematical concepts such as the Fourier Transform, these technologies offer precise control over frequency content, allowing composers and engineers to shape sound in ways that significantly affect emotional perception. This connection between sound engineering and psychoacoustics can deepen our understanding of how music affects perception and emotion, enabling more intentional emotional design in audio production.

In the following section, we explore the specific applications of these technologies, examining how intentional modifications in harmonic content can be observed through spectral analysis. This approach will help visualize the significant differences in how the same instrument and pitch can appear based on how they are manipulated.

6.2 Applications of Harmonic Manipulation

Manipulating harmonic structures digitally allows sound designers, composers, and audio engineers to shape musical perception. Expanding on the theoretical principles discussed in section 6.1, this section explores how digital effects can alter the harmonic content of a note, separate from pitch and instrument, and how these alterations affect perceptual and emotional responses.

Figure 7 illustrates the spectral effects of digital audio processing on a recorded G4 note on an electric guitar. All audio samples were recorded in Logic Pro and analyzed using Maztr's spectrum analyzer. Each spectrogram represents the same pitch under different processing conditions, including unprocessed signal (Figure 7a), reverb (Figure 7b), high-pass filter (Figure 7c), and distortion (Figure 7d). These transformations were chosen to highlight how changes in harmonic content affect both timbre and emotional perception.

Spectral Comparison of a G4 Note Under Various Audio Effects

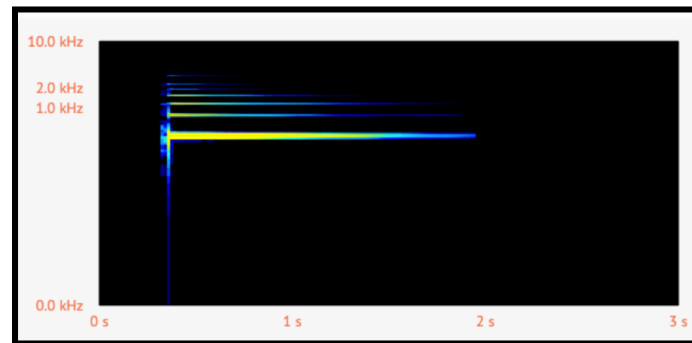


Figure 8a. Original G4 note showing a clean harmonic structure

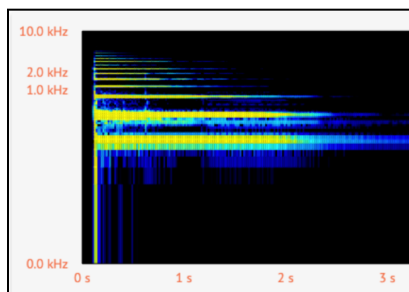


Figure 8b. Reverb-Processed signal

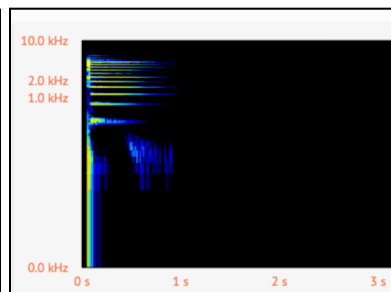


Figure 8c. High-Pass filtered signal

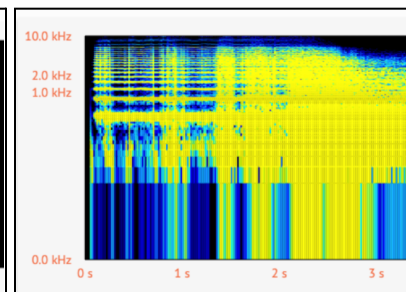


Figure 8d. Distorted signal

Digital audio effects significantly alter a sound's harmonic structure, thereby shaping its perceived timbre and emotional impact. The clean G4 note (Figure 7a) displays a stable harmonic pattern, with evenly spaced overtones above the fundamental frequency. When reverb is applied (Figure 7b), harmonics persist and decay gradually, emulating the sound reflections that occur in a physical space. This extended decay creates a sense of depth, and emotions often associated with calmness or introspection (Briand et al., 2020). In contrast, the high-pass filtered signal (Figure 7c) removes lower frequencies, making the harmonic structure brighter, producing a more fragile timbre, often used to highlight texture or clarity

(Marozeau et al., 2003). The distorted version (Figure 7d) adds dense inharmonic content, saturating the spectrum and disrupting harmonic clarity. This aggressive timbre often evokes emotional tension, depending on the musical context (Schellenberg & von Georgi, 2011).

These transformations reveal how intentional harmonic manipulation can affect the perceived qualities of sound, influencing the emotional interpretation of music. By understanding the mathematical and perceptual impacts of each effect, audio engineers can shape sound to evoke specific emotions or textures. While harmonic texture is fundamentally shaped by choices in instrumentation, arrangement, and performance technique, digital effects provide an additional layer of expressive control. This combined approach is especially crucial in professional sound design across various genres, from ambient to metal, where both musical artistry and audio processing contribute to emotional impact (Hailstone et al., 2009).

Additionally, a study by Disch and Edler (2008) explores how changes in harmonic and spectral properties through vocoders, an audio processing tool that analyzes the spectral characteristics of a sound and uses that information to modulate another sound source, can consistently trigger particular emotional responses. For example, spectral modifications such as harmonic enhancement, which emphasizes overtone alignment, can result in sounds perceived as bright and energetic, while inharmonic saturation, which introduces dissonant frequencies, often produces darker and more tense sounds. This research provides valuable insights regarding the connection between emotion and technology, highlighting the growing use of harmonic design to algorithmically recreate sounds tailored to illicit specific emotions.

Overall, understanding harmonic content and its manipulability allows artists and engineers to expand music-making and sound-editing techniques. Every choice in the digital audio world, whether it involves minor EQ adjustments or major spectral changes, can be ultimately explained through the basic principles of sound physics and the psychological reactions to it.

7. Synthesis of Interdisciplinary Insights and Final Analysis

7.1 Interdisciplinary Insights

Earlier sections of this paper established a multidisciplinary foundation on how we can understand the emotional impact of music, starting with the physics of sound, then understanding harmonic structures, auditory mechanics, the limbic system, reward systems, music perception as a subjective experience, and concluding with digital applications. The key to understanding the relationship between harmonic structures and human emotional responses lies in how physical principles, specifically wave behavior, frequency, and harmonic decomposition, with the help of mathematical tools, create the underlying structure through which the brain interprets music as emotionally meaningful.

Music is often defined as “organized sound,” a phrase famously attributed to composer Edgard Varèse (as cited in Schwartz & Childs, 1967). This idea captures the music as both a physical and perceptual phenomenon, where structure is what makes music so important. In this context, the way frequencies interact through harmonic relationships represents that organized structure. The human perception of music relies heavily on this concept. The frequencies that make up musical sounds are arranged into harmonic patterns using the same physical principles that describe wave behavior. Fourier analysis allows for the mathematical breakdown of sound into its basic sine wave components, similar to how the cochlea

biologically differentiates between frequencies along the basilar membrane. This mathematical breakdown of sound represents the foundation of a series of processes that involve perceptual and emotional responses to music.

Harmonic patterns like octaves, perfect fifths, and complex chords are not just properties of the sounds produced by instruments. They also play an important role in how our brains process and understand music. The auditory system uses a predictive approach to recognize and categorize relationships between different frequencies, guided by these harmonic principles. They are interpreted by the brain through an efficient, predictive framework. Our auditory system is naturally designed to identify, categorize, and assign meaning to these harmonic patterns, leading to a sense of recognition and expectation. When these anticipations are met, especially through harmonic resolution, the brain stimulates emotional pathways such as the limbic system and the reward system, releasing neurochemicals like dopamine, which are responsible for triggering pleasure in response to survival mechanisms or, in this case, structure and predictability within music.

Therefore, physics can then be understood as not just a descriptive tool for sound, but rather a mechanism of emotional structure. It allows for the understanding of *why* certain sound combinations evoke powerful emotional responses and how the auditory system can analyse musical sounds with varying levels of harmonic complexity.

The same principles that sound engineers and music producers use to analyze sound, through the use of DAWs, plug-ins, and digital instruments, all of which heavily rely on the Fourier series, frequency ratios, and amplitude decay, also link to processes that occur biologically within us. As a result, understanding music through physics does not mean seeing music as merely numbers or mathematical formulas. Instead, it highlights the deep connection between physical laws, mathematical frameworks, and human emotions. The interdisciplinary nature of this investigation suggests that the emotional impact of music is intertwined with the physics underlying it. This reinforces the idea that our emotional connection to music is not separate from physics, but rather, it emerges through it.

7.2 How Physics and Math-Based Research Can Deepen Our Understanding of Music and Emotion

While the connection between sound waves and emotion may not be evident at first glance, the framework provided by Fourier Analysis and harmonic theory continues to offer new pathways for exploration, especially in the field of psychoacoustics. As technology evolves, so does our ability to model, simulate, and predict the emotional effects of music using increasingly refined tools grounded in acoustics, physics, signal processing, and computational models.

One promising direction involves the integration of machine learning algorithms trained on Fourier-derived features, which could help identify which harmonic structures and timbral changes trigger specific emotional responses. For example, recent work in real-time music arrangement systems demonstrates how machine-learning models can manipulate harmonic and spectral content to evoke consistent emotional reactions across listeners (Wang et al., 2023). This research establishes a theoretical foundation that links harmonic decomposition and emotional responses to sound, bridging mathematical sound models and human perception. Future applications may involve personalized therapeutic music tailored to listeners' emotional profiles and real-time composition software that creates emotionally

targeted music using spectral and harmonic templates, as explored in systems like REMAST, which adapts musical arrangements to match emotional inputs through machine learning (Wang et al., 2023).

Additionally, mathematical modeling of musical expectations, such as tension and resolution cycles, could improve how artificial intelligence understands musical form. Expanding upon concepts like phase changes, amplitude decay, and spectral richness tracking, future research may help us further understand how variations in these elements correlate with emotional peaks in listener experiences, especially at a personalized level rather than a generalized one. This would not only have implications in music creation but also for enhancing human-computer interaction through more efficient sound design tools and techniques.

As we deepen our understanding of harmonic structures and how we perceive sound, the development of more advanced and sophisticated audio effects becomes increasingly possible. Innovations in frequency manipulation, especially in mastering, where precise control and understanding over the frequency spectrum are crucial, could lead to an improved understanding of how balance and emotional responses can be achieved through sound production. These advancements could become the foundations for the next generation of music producers and sound engineers, directly linking with the type of music that will be made in the future. By building on concepts such as Fourier analysis and auditory modeling, future technologies could not only refine sound quality but also provide producers with more deliberate control over their music's emotional and perceptual effects.

8. Conclusion

Despite the scientific approach of this investigation, it is important to acknowledge the inherent subjectivity of music perception. While harmonic structures can be analyzed mathematically and their effects can be examined through biological systems, the emotional response they trigger differs from person to person. Various factors, such as personal memories, cultural background, previous musical exposure, and neurological sensitivity, all influence how music is interpreted. These factors present natural limitations to fully quantifying music's emotional power, yet they also highlight the complexity and richness of this interdisciplinary subject.

This paper explores the essential question: *How do sound waves and their harmonic frequencies influence human emotional responses to music?*

Through an analysis of harmonic structures, auditory perception, and Fourier-based sound decomposition, it becomes clear that the emotional experience of music is deeply tied to fundamental physical concepts. From the organization of frequencies to the role of the brain's auditory and reward systems, physics and mathematics provide the structural framework through which sound is both processed and perceived. In particular, the role of Fourier analysis reveals a connection between the physical decomposition of sound waves and the brain's auditory processing mechanisms, suggesting that the way we interpret music emotionally may, in part, reflect a shared mathematical foundation. While this connection does not provide a definitive answer, it offers promising pathways for future interdisciplinary research into the physical foundations of music's emotional influence.

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